HBY-1009 Seat No.________

B. Sc. (Sem. IV) Examination

April / May – 2015

Mathematics : CC-MATH - 401

(Advanced Calculus)

Time : 3 Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) The figures to the right indicate the marks of corresponding questions.

1 (a) Obtain the formula of radius of curvature of the curve $p = f(r)$. **6**

OR

(a) Obtain the formula of radius of curvature of the curve $y = f(x)$. **6**

(b) Attempt any **two** : **12**

(1) Find the radius of curvature of the curve $y = \frac{C}{2}\left(e^{x/c} + e^{-x/c}\right)$.

(2) If ρ_1 and ρ_2 are the radii of curvature at the extremities of the focal chord of parabola $\frac{a}{r} = 1 + \cos\theta$ then prove that $\frac{1}{\rho_1^{2/3}} + \frac{1}{\rho_2^{2/3}} = \frac{1}{a^{2/3}}$.

(3) Find the radius of curvature of curve $x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$.

2 (a) Let $f : S \subset R^2 \to R$ be a continuous function on S which is bounded by $x = a$, $x = b$, $y = \phi(x)$, $y = \psi(x)$, where ϕ and ψ are continuous functions on $[a, b]$ such that $\psi(x) < \phi(x)$, $\forall x \in [a, b]$ then show that **6**

$$\iint_S f\,dxdy = \int_a^b \int_{\psi(x)}^{\phi(x)} f\,dxdy$$

OR

(a) Prove that $\Gamma n \, \Gamma\left(n + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}} \cdot \Gamma 2n$. **6**

(b) Attempt any two : **12**

(1) Change the order of integration of the double integration

$$\int_0^{2a} \int_{y^2/4a}^{3a-y} f\,dydx$$

(2) Evaluate : $\int_0^C \frac{x\,dx}{\sqrt{C^2 - x^2}}$

(3) By transforming into polar co-ordinates evaluate

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \left(x^2 + y^2\right) dxdy$$

3 (a) State and prove Green's theorem. **6**

OR

(a) Prove that **6**

$$grad\left(\overline{f}\cdot\overline{g}\right)=\overline{f}\times curl\ \overline{g}+\overline{g}\times curl\ \overline{f}+\overline{f}\cdot(\nabla\overline{g})+\overline{g}\cdot\left(\nabla\overline{f}\right)$$

(b) Attempt any two : **12**

(1) If $\overline{f}=\left(x^3,y^3,z^3\right)$ and $\overline{\gamma}=(x,y,z)$ then

prove that $cur\ \overline{f}=0$, $grad\left(div\ \overline{f}\right)=6\overline{r}$,

$\nabla\left(div\ \overline{f}\right)=18$.

(2) Verify Green's theorem $\oint_C\left(y^2\,dx+x\,dy\right)$.

where C is the boundary of the square whose vertices are (0, 0), (2, 0), (2, 2), (0, 2).

(3) Verify divergence theorem

$$\iint_S\left[\left(x^3-yz\right)dydz-2x^2y\,dzdx+z\,dxdy\right]$$

where S is the surface of the cube with faces $x=0$, $x=a$, $y=0$, $y=a$, $z=0$, $z=a$.

4 Attempt any **two** : **8**

(1) Obtain the formula of P-the length of perpendicular drawn from pole to a tangent to the curve $r = f(\theta)$.

(2) Prove that

$$\int_a^b (x-a)^{m-1} \cdot (b-x)^{n-1} dx = (b-a)^{m+n-1} \beta(m,n)$$

(3) Prove that

$$div\left(\bar{f} \times \bar{g}\right) = \bar{g} \cdot curl\ \bar{f} - \bar{f} \cdot curl\ \bar{g}$$

5 Attempt any **two** : **8**

(1) Find the p-r equation of the curve $r = a(1+\cos\theta)$.

(2) Evaluate : $\int_0^1 \int_x^{2x} x^2 y\, dxdy$

(3) Evaluate : $\int_0^\infty \frac{x^7\left(1-x^9\right)}{(1+x)^{25}} dx$

(4) Evaluate $\oint_C ((x+y)dx + (x-y)dy]$

where C is the boundary of region bounded by circle $x^2 + y^2 = 1$, line segments $y = 0$, $0 \le x \le 1$ and $x = 0$, $0 \le y \le 1$.

HBY-1012 Seat No.________

B. Sc. (Sem. IV) Examination

April / May – 2015

CC-MATH - 402 : Mathematics

(Advanced Linear Algebra)

Time : 3 Hours] [Total Marks : **70**

Instructions : (1) There are four questions and all questions are compulsory.
(2) Figures in the right side indicate the marks of question.

1 (a) Prove that **8**
A square matrix A is invertible if and only if the corresponding linear transformation T is non-singular.

OR

(a) If A and B are $n \times n$ invertible matrices then prove that AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$. **8**

(b) Attempt any two : **12**

(1) Solve the system of equations :

$$x - y + z = 0$$
$$2x + y - 3z = 0$$
$$-x + y + 2z = -1$$

(2) Determine the rank of matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \end{bmatrix}$$

by row reduction method.

(3) If $A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ and

$B_1=\{(1,1,1),(1,0,0),(0,1,0)\}$

$B_2=\{(1,2,3),(1,-1,1),(2,1,1)\}$

are ordered basis of vector space R^3 then find a linear transformation $T:R^3\to R^3$ such that $A=[T:B_1,B_2]$.

2 (a) Define Bilinear form which of the following f defined R^2 are bilinear form ? 8

(i) $f(\overline{x},\overline{y})=x_1y_2-x_2y_1$

(ii) $f(\overline{x},\overline{y})=(x_1-y_1)+x_2y_2$

where $\forall\ \overline{x}=(x_1,x_2)$ and $\overline{y}=(y_1,y_2)\in R^2$.

OR

(a) Let V be a n-dimensional vector space and $B=\{x_1,x_2,\dots\dots\dots x_n\}$ be an ordered basis of V then for any ordered set $S=\{a_1,a_2,\dots\dots,a_n\}$ of n-scalars $\exists$ a unique linear functional f on V such that $f(x_i)=a_i$ where $i=1,2,\dots\dots\dots,n$. 8

(b) Attempt any **two** : **12**

(1) Find dual basis from the basis $\{(1,-1,3),(0,1,-1),(0,3,-2)\}$.

(2) If ϕ and ψ are two bilinear form on vector space $v(k)$ and $f : v \times v \to k$ defined $f(u,v) = \phi(u) \cdot \psi(v)$, then show that f is a bilinear form on v.

(3) By using Gramm Schmitz's process obtain the orthonormal basis from the basis $\{(-1,1,1),(1,-1,1),(1,1,-1)\}$.

3 (a) State and prove Cayley-Hamilton theorem. **8**

OR

(a) Prove that eigen vector of symmetric linear map corresponding to different eigen values are perpendicular to each other.

(b) Attempt any **two** : **12**

(1) Find the minimal polynomial for matrix

$$A = \begin{bmatrix} -4 & -4 & 5 \\ 7 & 4 & -1 \\ 5 & 7 & -4 \end{bmatrix}$$

(2) Find eigen values and corresponding eigen vectors of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(3) Find characteristic equation of matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

4 Attempt any **three** : **10**

(1) State and prove Schwartz's inequality.

(2) Define :

(i) Inner product space

(ii) Eigen values and Eigen vectors

(iii) Minimal polynomial.

(3) Prove that the orthogonal set of vectors in innerproduct space is linearly independents.

(4) If $T : R^3 \to R^2$

$T(e_1) = (1, -1),\ T(e_2) = (-2, 2),\ T(e_3) = (3, 5)$

$B_1 = \{(1, -1, 0),\ (0, 1, -1),\ (-1, 0, 1)\}$

$B_2 = \{(-1, 1), (-2, 5)\}$ are basis of R^3 and R^2 respectively. Then find $[T : B_1, B_2]$

(5) If T is invertible and λ is an eigen value of T, then show that λ^{-1} is eigen value of T^{-1}.

HBY-1026-33 Seat No.________

B. Sc. (Sem. IV) Examination

April / May – 2015

(1) Mathematics : ES-22 (Business Mathematics - IV)
(2) ESMATH-32 : Business Mathematics-II (Elective)

Time : **3** Hours] [Total Marks : **50**

(1) Mathematics : ES-22 (Business Mathematics - IV)

1 Attempt any five : **25**

(1) In a group of people, 28 like Gujarati movies, 30 like Hindi movies, 42 like English movies, 5 like both Gujarati and Hindi movies, 8 like Hindi and English movies, 8 like Gujarati and English movies and 3 like Gujarati, Hindi and English movies, what is the least number of people in the group ?

(2) Prove that $p \vee q = p \vee r$ or $p \wedge q = p \wedge r$ then $q = r$. And also prove that Negation is uniquely.

(3) State and prove De-Morgan's laws.

(4) $A \subset U$ and $B \subset U$, such that $n(A)=20$, $n(B)=30$, $n(U)=100$, $n(A \cap B)=10$ then find $n(A' \cap B')$.

(5) Show that the following statement are true by

(i) Direct Method

(ii) Contra positive method.

If $x^5+16x=0$, then $x=0$.

(6) Construct the Truth table.

(i) $(p \wedge q) \vee r$

(ii) $(p \vee q) \wedge r$

(iii) $[(\sim p) \vee (\sim q)] \wedge r$

2 (a) Attempt any four : **16**

(i) In how many ways can the letters of the word PERMUTATIONS be permuted ? Also find,

(a) How many start with P and end in S?

(b) In how many of them vowels are together ?

(ii) A reception committee consisting of 6 students for the annual function of a school is to be formed form 8 boys and 5 girls. In how many ways can we do it if the committee is to contain (i) Exactly 4 girls (ii) At most 2 Girls ?

(iii) How many arrangements can be made with the letters of the word MATHEMATICS and in how many of them vowels occurs together ?

(iv) In how many ways can seven digit numbers greater than 10,00,000 be formed using digits 1,2,0,2,4,2,4 ?

(v) If $\binom{n}{r-1}=36$, $\binom{n}{r}=84$, $\binom{n}{r+1}=126$, find n and r.

(b) Attempt any three : **9**

(i) Find r, If $5\cdot {}_4P_r = 6\cdot {}_5P_{r-1}$.

(ii) Prove $\binom{2n}{n}=\dfrac{2^n\left[1\cdot3\cdot5\cdot7........(2n-1)\right]}{n!}$.

(iii) If m vertical bars meet n horizontal bars, how many rectangles will be formed ?

(iv) How many three digit numbers are there which are multiples of 5 ?

(Without repetition of digits)

(2) ESMATH-32 : *Business Mathematics-II (Elective)*

Instructions : (1) All questions are compulsory.

(2) Figures to the right indicate marks of the corresponding question.

1 (a) Attempt any two : **10**

(i) The probability (i) that A can solve a problem in statistics is 4/5 (ii) that B can solve it is $\frac{2}{3}$ (iii) that C can solve it is $\frac{3}{7}$. If all of them try independently, find the probability that the problem will be solved.

(ii) Explain : Statistical definition of probability and mutually exclusive events.

(iii) Two players A and B toss an unbiased die alternatively. He who first throws a six wins the game. If A begins, what is the probability that B wins the game ?

(b) Attempt any two : **10**

(i) If A and B are two events such that

$$P(A) = \frac{2}{3},\ P(\bar{A} \cap B) = \frac{1}{6} \text{ and } P(A \cap B) = \frac{1}{3}.$$

Find P(B), $P(A \cup B)$, P(A/B) and $P(\bar{A} \cap \bar{B})$.

(ii) Two students X and Y work independently on a problem. The probability that X will solve it is 3/4 and the probability that y will solve it is 2/3. What is the probability that the problem will be solved.

(iii) A box contains 4 identical dice out of which three are fair and the fourth is loaded in such a way that the face marked as 5 appears in 60% of the tosses. A die is selected at random from the box and tossed. If it shows 5, what is the probability that it was a loaded die ?

(c) Answer the following : 5

(i) 5 red and 2 black balls, each of different sizes are randomly laid down in a row. Find the probability that the two end balls are black.

(ii) Prove that: $P(A') = 1 - P(A)$, for an event A.

(iii) State : Baye's theorem.

(iv) Find the probability of throwing a total of six in a single throw with two unbiased dice.

(v) State conditional probability theorem.

2 (a) Attempt any two : **10**

(i) Assuming that it is true that 2 in 10 industrial accidents are due to fatigue, find the probability that

(a) Exactly 2 of 8 industrial accidents will be due to fatigue

(b) At least 2 of the 8 industrial accidents will be due to fatigue.

(ii) Define the Binomial distribution and discuss its applications.

(iii) The scores made by candidate in a certain test are normally distributed with mean 500 and S.D. 100 what percent of candidates receive scores 600 ?

(b) Attempt any two : **10**

(i) A book contains 100 misprints distributed randomly throughout its 100 pages. What is the probability that a page observed at random contains at least two misprints. Assume Poisson distribution.

(ii) Discuss the probability distribution of a discrete random variable.

(iii) If it rains, an umbrella salesman earns Rs. 100 per day. If it is fair, he loses Rs. 15 per day. What is his expectation if the probability of rain is 0.3 ?

(c) Answer the following : 5

(i) Write the probability mass function of normal distribution.

(ii) Is there any fallacy in the statement "the mean of a Binomial distribution is 20 and its S.D. 7".

(iii) A coin is tossed six times. What is the probability of obtaining 6 heads ?

(iv) If mean of a poisson distribution is 4 then find its S.D.

(v) Define mean in Binomial distribution.
