SIRP.T.SCIENCECOLLEGE,MODASA

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Certificate

This is to certify that the following students of B.Sc.(Sem-IV) has successfully completed the project entitled **History of Reimann-Steltjes Integration and its properties** under the guidance of Prof. A. J. Bhavsar, Assistant Professor, Department of Mathematics, SIR P. T. SCIENCE COLLEGE, MODASA during the year 2022-2023.

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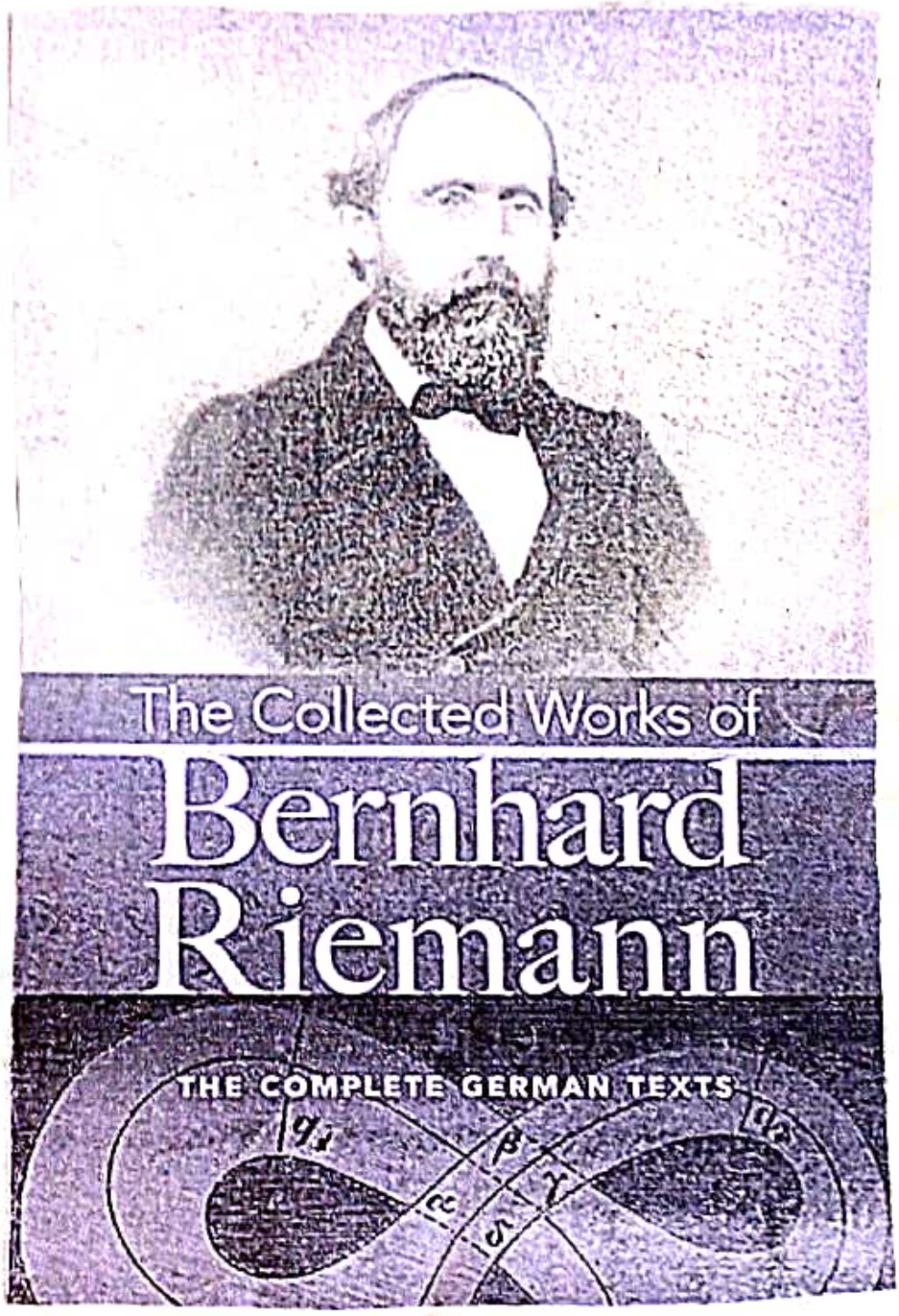
THE RIEMANN STIELJES INTEGRAL

INDEX

The language of friendship is not words but meanings.



The greatest strategy is
omed if it's implemented badly."
- Bernhard Riemann

Bernhard Riemann

Born	Georg Friedrich Bernhard Riemann 17 September 1826
Died	20 July 1866
Fields	Mathematics - physics
Institution	University of Gottingen
Thesis	Grundlagen für eine allgemeine Theorie der Funktionen einer veränderlichen Complexen
Nationality	German

The language of friendship is not words but meanings.

FRIENDS

Riemannian geometry

Riemann found the correct way to extend into n-dimensions the differential geometry of surfaces, which gauss himself proved in his theorema egregium. The fundamental objects are called the Riemannian metric and the Riemann curvature tensor.

Complex analysis

He established a geometric foundation for complex through Riemann surfaces, through which multi-valued functions like logarithm or the square root could become one-to-one functions. Complex functions are harmonic functions on these surfaces and are described by the location of their singularities and the topology of the surfaces. The topological "genus" of the Riemann surfaces is given by $g = w/2 - n + 1$, where the surface has n leaves coming togethes at w branch points. For $g > 1$ the Riemann surface has $(3g-3)$ parameters.

FRIENDS

Real analysis

In the field of real analysis, he discovesd the Riemann integral integral in his habilitation. He showed that every piecewise continuous function is integrable.
Stieltjes integral goes back to the gottinges mathematician, so they are named togethes the Riemann-Stieltjes integral.

Numbes theory

Riemann made some famous contribution to modern analytic numbes theory.

Riemann renew of pafnuty chebyshev's wosk on the prime numbes theorem. He had visited Disichlet in 1852

Page :
Date :

*. Definition and existance of the integral :-

Let f and α be bounded functions on $[a, b]$ and α be monotonic increasing on $[a, b]$, $b \geq a$.

corresponding to any partition

$P = \{a = x_0, x_1, \ldots, x_n = b\}$, of $[a, b]$

we write

$\Delta\alpha_i = \alpha(x_i) - \alpha(x_{i-1})$, $i = 1, 2, \ldots, n$.

It is clear that $\Delta\alpha_i \geq 0$.

$$U(P, f, \alpha) = \sum_{i=1}^{n} m_i \Delta\alpha_i$$

$$L(P, f, \alpha) = \sum_{i=1}^{n} m_i \Delta\alpha_i$$

where m_i, M_i, are the bounds (infimum and supremum respectively) of f in Δx_i, respectively called the upper and the lower Sums of f corresponding to the partition P.

If m, M are respectively the lower and the upper bounds of f on $[a, b]$, we have

$$m \leq m_i \leq M_i \leq M$$

$$\Rightarrow m\Delta x_i \leq m_i \Delta x_i \leq M_i \Delta x_i \leq M \Delta x_i, \quad \Delta x_i \geq 0$$

Putting $i = 1, 2, \ldots, n$ and adding all inequalities, we get

$$m\{\alpha(b) - \alpha(a)\} \leq L(P, f, \alpha) \leq U(P, f, \alpha) \leq M\{\alpha(b) - \alpha(a)\}$$

As in Riemann integrations, we define two integrals, which always exist by a similar reasoning.

$$\overline{\int_a^b} f\,d\alpha = \inf U(P, f, \alpha)$$

$$\underline{\int_a^b} f\,d\alpha = \sup L(P, f, \alpha)$$

the infimum and supremum being taken over all partitions of $[a, b]$. These are respectively called the upper and the lower integrals of f with respect to α.

These two integrals may or may not be equal. In cases these two integrals are equal,

i.e.

$$\overline{\int_a^b} f\,d\alpha = \underline{\int_a^b} f\,d\alpha,$$