SIRP.T.SCIENCECOLLEGE,MODASA

(Managed by THE M. L. GANDHI HIGHER EDUCATION SOCIETY)

Certificate

This is to certify that the following students of B.Sc.(sem-IV) has successfully completed the project entitled **Continuity and Homomorphism Properties of Topological Spaces** under the guidance of Dr. V. R. Patel, Head and Assistant Professor, Department of Mathematics, SIR P. T. SCIENCE COLLEGE, MODASA during year 2022-2023.

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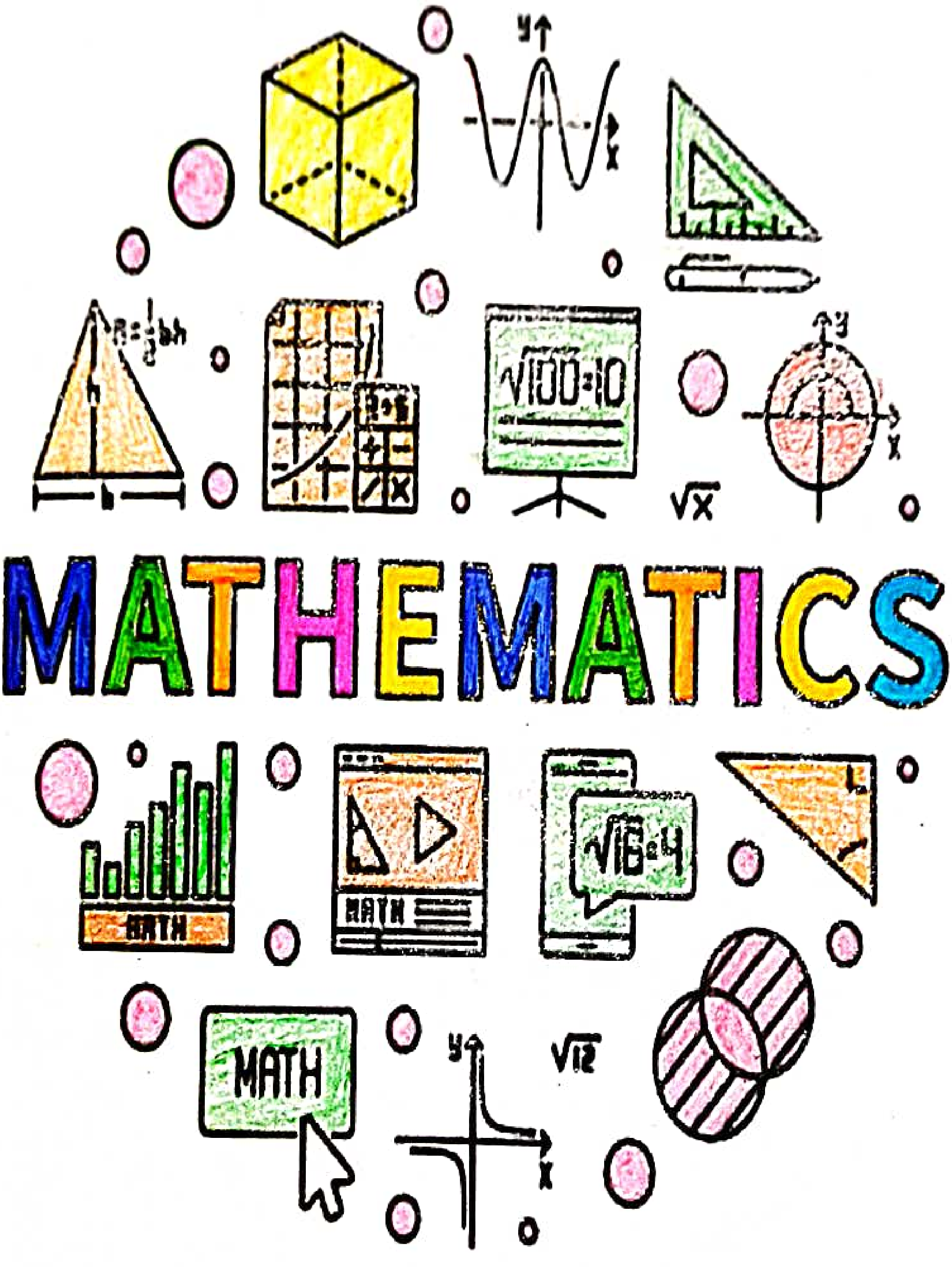
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Interior Point

* Defination :

Let $(X, \mathfrak{J})$ be a topological space and $A \subset X$. Then a point $x \in A$ is said to be the interior point of A iff A is nbhd of x.

i.e. iff $\exists$ an open set $G \ni x \in G \subset A$

⟶ x is an interior point of A iff A is nbhd of x.

⟶ x is an interior point of A iff $\exists$ an open set G containing $x \ni G \cap A' = \phi$

⟶ Interior point is denoted as A° or A' or $i(A)$ or $\mathrm{int}(A)$.

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Examples.

1). $A = [0,1] \cup \{2\}$.

$\Rightarrow A^{\circ} = (0,1)$

2). $A = [0,1] \cup [2,3]$

$\Rightarrow A^{\circ} = (0,1) \cup (2,3)$

3) Let $X = \{a,b,c,d\}$ and
$\mathfrak{I} = \{\phi, X, \{a,c,d\}, \{b,d\}, \{d\}\}$ if $A = \{b,d\}$
then find closure of A and interior of A.

Remark :

It is clear from the definition that every interior Point of A is in A

i.e. $A^{\circ} \subset A$.

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Results on Interiors of sets.

* **Theorem:**

Let $(X, \mathfrak{J})$ be a topological space and let $A \subseteq X$. Then A° is the union of all open subsets of A

* **Theorem:**

Let $(X, \mathfrak{J})$ be a topological space and let $A \subseteq X$ Then.

(1) A° is the largest open subset of A

(2) A is open iff $A^{\circ} = A$.

* **Theorem:**

Let $(X, \mathfrak{J})$ be a topological space and let $A \subseteq$

Then, (i) A° equals the set of all

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those points of A, which are not the limit points of A°

(ii) $A^{\circ} = \{(\overline{A^c})\}^c$

(iii) $\overline{A} = \{(A^c)^{\circ}\}^c$

* Theorem:

Let $(X, \mathfrak{I})$ be a topological space and let A, B be any subsets of X. Then,

i) $\phi^{\circ} = \phi$

ii) $X^{\circ} = X$

iii) $A \subseteq B \Rightarrow A^{\circ} \subseteq B^{\circ}$

iv) $A^{\circ} \cup B^{\circ} \subseteq (A \cup B)^{\circ}$

v) $(A \cap B)^{\circ} = (A^{\circ} \cap B^{\circ})$

vi) $(A^{\circ})^{\circ} = A^{\circ}$.

Lemma:

Given a subset A of a topological space $(X, \mathfrak{I})$ and an open set O contained in A then

$O \subseteq Int(A)$

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⇒ Lemma :

Given a subset A of a topological space X if $x \in Int(A)$ then $x \in O$, for some open set O such that $O \subset A$.

⇒ Lemma :

If $\{O_\alpha\}_{\alpha \in I}$ is the family of all open sets contained in A then

$Int(A) = \bigcup_{\alpha \in I} O_\alpha$

OR

Show that Int(A) is largest open set contained in A.

Remark :

In general, $(A \cup B)^\circ \neq (A^\circ \cup B^\circ)$

For example, on the real line $(\mathbb{R}, \mathcal{U})$, if we consider the sets.

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* Theorem.
The relation of homeomorphism on the set of all topological spaces, is an equivalence relation.

* Theorem:
Let $(X, \mathfrak{J}) \cong (Y, \mathfrak{J}')$ and let f be the corresponding homeomorphism. Then, f maps every isolated subset of X onto an isolated subset of Y.

* Theorem:
A homeomorphic image of a first countable space is first countable.

* Theorem:
A homeomorphic image of a second countable space is second countable.