SIRP.T.SCIENCECOLLEGE,MODASA

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Certificate

This is to certify that the following students of B.Sc.(Sem-IV) has successfully completed the project entitled **Basic Concept On LPP** under the guidance of Prof. A. J. Bhavsar, Assistant Professor, Department of Mathematics, SIR P. T. SCIENCE COLLEGE, MODASA during year 2022-2023.

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IMPORTANT DEFINITIONS

Solution: The set of values of decision variables x_j $(j = 1, 2, \ldots, n)$ that satisfy the constraints of an L.P problem is said to constitute the solution to that L.P problem.

Feasible solution: The set of values of decision variables x_j $(j = 1, 2, \ldots, n)$ that satisfy all the constraints and non-negativity conditions of an L.P problem simultaneously is said to constitute the feasible solution to that L.P problem.

Infeasible solution: The set of values of decision variables x_j $(j = 1, 2, \ldots, n)$ that do not satisfy all the constraints and non-negativity conditions of an L.P problem simultaneously is said to constitute the infeasible solution to that L.P problem.

Basic solution: For a set of m simultaneous equations in n variables $(n > m)$ in an L.P problem, a solution obtained by setting $(n-m)$ variables equal to zero and solving for remaining m equations in m variables is called a basic solution of that L.P problem.

The (n-m) variables whose value did not appears in basic solution are called non-basic variables and the remaining m variables are called basic variables.

Basic feasible solution: A feasible solution to an L.P problem which is also the basic solution is called the basic feasible solution. That is, all basic variables assume non-negative values. Basic feasible solution is of two types:

(a) Degenerate A basic feasible solution is called degenerate if the value of at least one basic variable is zero.

(b) Non-dengenerate A basic feasible solutions is called non-dengenerate if value of all m basic variables is non-zero and positive.

optimum basic feasible solution: A basic feasible solution that optimizes (maximizes or minimizes) the objective function value of the given L.P problem is called an optimum basic feasible solution.

Unbounded solution: A solution that can increase or decrease infinitely the value of the objective function of the L.P problem is called an unbounded solution.

GRAPHICAL SOLUTION METHODS OF LP PROBLEM:

while obtaining the optimal solution to the LP problem by the graphical method, the statement of the following theorems of linear programming is used.

- The collection of all feasible solutions to an LP problem constitutes a convex set whose extreme points correspond to the basic feasible solutions.

- There are a finite number of basic feasible solution space

- If the convex set of the feasible solutions of the system of simultaneous equations $Ax = b$. $x \geq 0$, is a convex polyhedron, then at least one of the extreme points gives an optimal solution.

• If the optimal solution occurs at more then one extreme point, the value of the objective function will be the same for all convex combinations of these extreme points.

Remarks:

1. A convex set is a polygon and by 'convex' we mean that if any two points of a polygon are selected arbitrarily, a straight line segment joining these two points lies completely within the polygon.

2. Each corner (extreme or vertex) point of the feasible region (space or area) falls at the intersection of two constraint equalities.

3. The extreme points of the convex set provide the basic feasible solution to the LP problem.

3.3.1 Extreme point solution Method:

In this method, the coordinates of all corner (or extreme) points of the feasible region (space or area) are deter

-mined and then value of the objective function at each of these points is computed and compared. The coordinates of an extreme point where the optimal (maximum or minimum) value of the objective function is found represent solution of the given L.P problem. The steps of the method are summarized as follows:

step 1: Develop an LP model state the given problem in the mathematical LP model as illustrated in the previous chapter.

step 2: plot constraints on graph paper and decide the feasible region

(a) Replace the inequality sign in each constraint by by an equality sing.

(b) Draw these straight lines on the graph paper and decide each time the area of feasible solutions according to the inequality sign of the constraint. shade the common portion of the graph that satisfies all the constraints simultaneously drawn so far.

(c) The final shaded area is called the feasible region (or solution space) of the given LP problem. Any point inside this region is called feasible solution and this provides values of x_1 and x_2 that satisfy all the constraints.

Step 3: Examine extreme points of the feasible solution space to find an optimal solution

(a) Determine the coordinates of each extreme point of the feasible solution space.

(b) Compute and compare the value of the objective function at each extreme point

(c) Identify the extreme point the gives optimal (max. or min.) value of the objective function.

4.3 SIMPLEX ALGORITHM (MAXIMIZATION CASE)

The steps of the simple algorithm for obtaining an optimal solution (if it exists) to a linear programming problem are as follows:

Step 2 : Formulation of the mathematical model :

(i) Formulate the LP model of the given problem.

(ii) If the objective function is of minimization, then convert it into equivalent maximization, by using the following relationship

Minimize $Z = -$Maximize Z^*,
where $Z^* = -Z$.

(iii) check whether all the b_i $(i = 1, 2, \ldots, m)$ values are positive. If any one of them is negative, the multiply the corresponding constraint by -1 in order to make $b_i \geq 0$. In doing so, remember to change a $\leq$ typy constraint to a $\geq$ type ~~Cons~~ constraint, and vice versa.

(iv) Express the given LP problem in the standard form by adding additional variables in constraints (as per requirement) and assign a zero-cost coefficient to these variables in the objective function.

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(v) Replace each unrestricted variable

(if any) with the difference of the two non-negative variables.

step 2 : set-up the initial solution :-

write down the coefficients of all the variables in the LP problem in a tabular from, as shown in Table 4.2, in order to get an initial basic feasible solution $[x_B = B^{-1}b]$.

4.4 SIMPLEX ALGORITHM (MINIMIZATION CASE)

In certain cases, it is difficult to obtain in initial basic feasible solution of the given LP problem. such cases arise

(i) when the constraints are of the $\leq$ type.

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad x_j \geq 0$$

and value of few right-hand side constraints is negative [i.e. $b_i < 0$]. After adding the non-negative slack variable s_i ($i = 1, 2, \ldots, m$), the initial solution so obtained will be $s_i = -b_i$ for a particular resource, i. This solution is not feasible because it does not satisfy non-negativity conditions of slack variables (i.e. $s_i \geq 0$).

(ii) when the constraints are of the $\geq$ type.

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i, \quad x_j \geq 0$$

After adding surplus (negative slack) variable s_i the initial solution so obtained will be $-s_i = b_i$ or $s_i = -b_i$

$$\sum_{j=1}^{n} a_{ij} x_j - s_i = b_i , \quad x_j \geq 0, s_i \geq 0$$

This solution is not feasible because it does not satisfy non-negativity conditions of surplus variables (i.e. $s_i \geq 0$). In such a case, artificial variables A_i ($i = 1, 2, \dots, m$) are added to get an initial basic feasible solution. The resulting system of equations then becomes:

$$\sum_{j=1}^{n} a_{ij} x_j - s_i + A_i = b_i$$

$$x_j, s_i, A_i \geq 0 , \quad i = 1, 2, \dots, m$$

These are m simultaneous equations with $(n+m+m)$ variables (n decision variables, m artificial variables and m surplus variables). An initial basic feasible solution of LP problem with such constraints can be obtained by equating $(n+2m-m) = (n+m)$ variables equal to zero. Thus the new solution to the given LP problem is: $A_i = b_i$ ($i = 1, 2, \dots m$), which is not the solution to the original system of equations because the two systems of equation are not equivalent.

Thus, to get back to the original problem, artificial variables must be removed from the optimal solution. There are two methods for removing artificial variables from the solution:

- Two-phase method
- Big-M method or method of penalties.

Remark :

Artificial variables have no meaning in a physical sense and are only used as a tool for generating an initial solution to an L.P.P. Before the optimal solution is reached, all artificial variables must be dropped out from the solution mix. This is done by assiging coefficients to these variables in the objective function. These variables are added to those constraints with equality (=) and greater than or equal to ($\geq$) sign.

Two-phase method :- In the first phase of this method, the sum of the artificial variables is minimized subject to the given constraints in order to get a basic feasible Solution of the L.P.P. The second phase minimizes the original objective function starting with the basic feasible solution obtained at the end of the first phase. Since the solution of the LPP is completed in two phases, this method is called the two-phase method.

Advantages of the method

1. No assumptions on the original system of Constraints are made, i.e. the system may be redundant, inconsistent or not solvable in non-negative numbers.

2. It is easy to obtain an initial basic feasible Solution for phase - I.

3. The basic feasible solution obtained at the end of phase - I is used as initial solution for phase II

Steps of the Algorithm : phase I

Step 1 :(a) - If all the constraints in the given L.P.P are 'less than or equal to' ($\leq$) type, then phase II can be directly used to solve the problem otherwise, the neccessary number of surplus & artificial variables are added to convert constraint into equality constraints.

Step 2 : Assign zero coefficient to each of the decision variables (x_j) and to the surplus variable' s_i and assign -1 coefficient to each of the artificial variables. This yields the following auxiliary LP Problem.

maximize $z^* = \sum_{i=1}^{m} (-1) A_i$

Subject to the constraints. $\sum_{i=1}^{n} a_{ij} x_j + A_j = b_i$, $i = 1, 2, \ldots m$

and $x_j, A_j \geq 0$

Step 3 : Apply the simplex algorithm to solve this auxiliary LPP. The following three cases may arise at optimality.

a) max $z^* = 0$ and at least one artificial variable is present in the basic with positive value. This means that no feasible solution exists for the original L.P.P.

5 b) max $z^* = 0$ and no artificial variable is present in this basic. This means that only decision

variables (a_j's) are present in the basis and hence proceed to phase II to obtain an optimal basic feasible solution on the original LPP.

c) max $z^* = 0$ and atleast one artificial variable is present in the basis at zero value. this means that a feasible solution to the auxiliary LPP is also a feasible solution to the original LPP. In order to arrive at the basic feasible solution, proceed directly to phase II or else eliminate the artificial basic variable and then proceed to phase-II.

Remark : Once an artificial variable has left the basis, it has served its purpose and can, therefore, be artificial from the simplex table. An artifial variable is never considered for re-entry into the basis.

Phase II : Assign actual coefficients to the varibles in the objective function and zero coefficient to the artificial variables which appear at zero value in the basis at the end of phase I. The last simplex table of phase I can be used as the intial simplex table for phase-II. Then apply the usual simplex algorithem to the modified simplex table in order to get the optimal solution to the original problem. Artificial variables that do not appear in the basis may be removed.

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